IV. Conclusion

We have presented a semi-analytical analysis of general multilayered multiconductor transmission lines with arbitrary cross section conductors using the M-strips model. This procedure has proved its ability to compute in a fast and accurate way the characteristic matrices of the analyzed transmission lines. It has been also shown how the M-strip model combined with the Wheeler's incremental inductance rule yields sufficiently accurate results for the conductor losses assuming strong skin effect. The studied examples have shown that rectangular conductors and even circular conductors can be efficiently modeled with a reasonable number of thin strips. This latter fact and the enhanced numerical treatment here applied suggests that our scheme may be used as a good basis for CAD of general transmission lines.

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A Numerical Method of Evaluating Electromagnetic Fields in a Generalized Anisotropic Medium

Hung-Yu Yang

Abstract—**A transition matrix method is commonly used to deal with the problems of either plane-wave scattering from or the Green's function of a generalized anisotropic medium. This method, although rigorous analytically, introduces numerical breakdown, when the layers are electrically thick and the waves are evanescent. A variable transformation method is developed to deal with the exponentially-growing terms associated with exponential-matrix method. The proposed scheme is suitable for the numerical analysis of generalized anisotropic layers including ferrites, magneto-plasmas, chiral layers, and bianistropic layers.**

I. Introduction

In the past, there have been considerable interest in the investigation of the interaction of electromagnetic waves with anisotropic materials. The classical formulation for antennas on layered media employing a combination of TE and TM vector potential functions limits the applications to isotropic or uniaxial media. In recent years, the interest in the technology of printed circuit elements on anisotropic substrates has stirred the investigation of electromagnetic waves interaction with generalized anisotropic layered media. A spectral exponential 4 × 4 matrix method has been developed to deal with embedded dipoles in or scattering from a layered generalized anisotropic structure [1]–[6]. The exponential matrix method is a useful numerical method in dealing with waves in media with arbitrary anisotropy. There, the derivation of analytic form of waves is often complicate and tedious if not impossible. Most published research in the area of electromagnetic waves in layered anisotropic media dealt with the analytic aspect of the problem. The full-wave numerical implementation of the spectral matrix method has been applied for microstrip transmission-lines [7]–[9] and for printed antennas [10], [11]. A critical step in the exponential-matrix method is to develop the transition matrices which relate the electromagnetic fields at one planar interface to the others. This method although elegant analytically has inherent deficiency in the numerical implementation. Problems arise when the wave numbers in the direction of inhomogeneity are complex-valued. If the layers are electrically thick enough, the transition matrices become numerically singular and can no longer pass the complete information of fields. The physical explanation is that from one layer interface to another, part of the waves die out before reaching the interface. The remaining propagating waves are degenerate.

As a result, the 4 × 4 transition matrix is singular. This problem is particularly serious in dealing with antennas and circuits on anisotropic media, where the plane wave representations of fields always include the evanescent plane wave spectrum. This numerical singularity (or overflow) problem occurs often in dealing with isotropic or uniaxial media, where the problem is overcome in the analytic formulation, by normalizing the variables such that we deal with the "$\tanh$" functions instead of the "$\cosh$" or "$\sinh$" functions.

In this paper, a scheme utilizing variable transformation is developed. The idea is to extract the large exponential terms in

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the formulation and transform them into the variables which are used to represent fields at each interface. This procedure ensure all the information of fields in one interface is effectively passed on to the next interface. The proposed numerical scheme and its numerical implementation is described in Section II. An example of the application is discussed in Section III.

II. Variable Transformation in Matrix Exponential Method

For the convenience of discussion, we consider the problem of a plane wave scattering from a planar (x-y plane) generalized anisotropic layer ($0 \leq z \leq d$) shown in Fig. 1. The approach for the problem with current sources is similar as will be seen shortly. The extension of the method to deal with multilayer generalized anisotropic media will be discussed elsewhere. In the spectral exponential matrix method, the x and y spectral field components in the anisotropic medium derived from Maxwell's curl equations with some algebraic manipulations become four coupled first-order differential equations [1]–[11] which in a matrix form are

$$\frac{\partial}{\partial z}[\tilde{\psi}(z)] = [A][\tilde{\psi}(z)] \tag{1}$$

where

$$[\tilde{\psi}(z)] = \begin{bmatrix} k_x\tilde{H}_x(z) + k_y\tilde{H}_y(z) \\ k_y\tilde{H}_x(z) - k_x\tilde{H}_y(z) \\ k_x\tilde{E}_x(z) + k_y\tilde{E}_y(z) \\ k_y\tilde{E}_x(z) - k_x\tilde{E}_y(z) \end{bmatrix}. \tag{2}$$

$\tilde{E}_x$, $\tilde{E}_y$, $\tilde{H}_x$, and $\tilde{H}_y$ are the Fourier transforms of the tangential field components and $[A]$ is a 4×4 matrix where the elements are functions of spectral variables k_x, k_y and material parameters. If one defines the 4×4 matrix $[\tilde{\phi}]$ as the eigenvector matrix with the eigenvalues λ_i, $i = 1, 2, 3, 4$ of $[A]$, the solution of (1) is

$$[\tilde{\psi}(d^-)] = [T(d)][\tilde{\psi}(0)] \tag{3}$$

where

$$[T(d)] = [\tilde{\phi}]\begin{bmatrix} e^{\lambda_1 d} & 0 & 0 & 0 \\ 0 & e^{\lambda_2 d} & 0 & 0 \\ 0 & 0 & e^{\lambda_3 d} & 0 \\ 0 & 0 & 0 & e^{\lambda_4 d} \end{bmatrix}[\tilde{\phi}]^{-1}. \tag{4}$$

The electromagnetic fields in the air ($z \geq d^+$ and $z \leq 0^-$) can be derived in a straightforward manner by the combination of TE and TM vector potential functions. This result can be shown as

$$[\tilde{\psi}(d^+)] = \begin{bmatrix} j\sqrt{k^2 - k_0^2}\,\tilde{a} \\ \omega\epsilon_0\tilde{b} \\ j\sqrt{k^2 - k_0^2}\,\tilde{b} \\ -\omega\mu_0\tilde{a} \end{bmatrix}$$

and

$$[\tilde{\psi}(0^-)] = \begin{bmatrix} -j\sqrt{k^2 - k_0^2}\,\tilde{c} \\ \omega\epsilon_0\tilde{d} \\ -j\sqrt{k^2 - k_0^2}\,\tilde{d} \\ -\omega\mu_0\tilde{c} \end{bmatrix} \tag{5}$$

where $\tilde{a}$, $\tilde{b}$, $\tilde{c}$ and $\tilde{d}$ are the quantities to be determined from the following equation

$$[\tilde{\psi}(d^+)] - [T(d)][\tilde{\psi}(0^-)] = [Q]_{\mathrm{inc}} \tag{6}$$

and $k = \sqrt{k_x^2 + k_y^2}$, where $[Q]_{\mathrm{inc}}$ is related to the incident plane wave. For the problem with a current source, the right-hand side of (6) should be the corresponding spectral current component. The exponential matrix method described above is rigorous analytically. However, in numerical implementation, this method often break-downs. Without the loss of generality, it is assumed that $\mathrm{Re}(\lambda_1) \geq \mathrm{Re}(\lambda_2) \geq \mathrm{Re}(\lambda_3) \geq \mathrm{Re}(\lambda_4)$. In many practical applications when $\mathrm{Re}(e^{\lambda_1 d}) \gg 1$, the transition matrix defined in (4) becomes numerically singular. As a result, the numerical inversion of the matrix equation in (6) provides erroneous results.

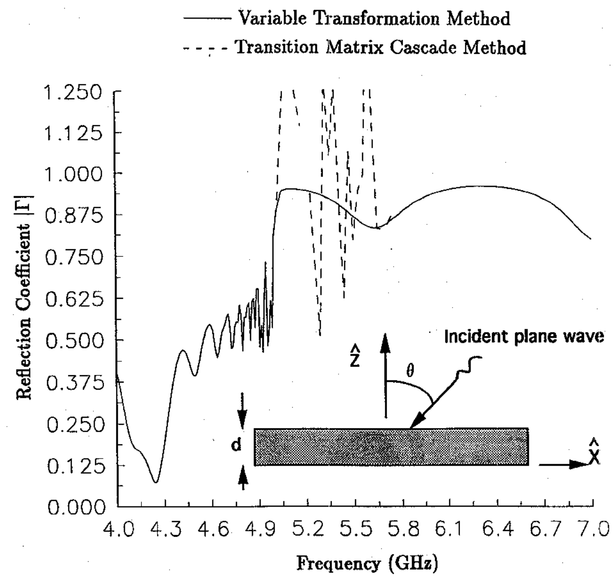

Fig. 1. Reflection from an in-plane biased ferrite layer. Biased field (H_0) 1000 Gauss in the $\hat{x}$ direction, magnetization: 2250 Gauss, TM incidence with $\theta_i = 30^\circ$ and $\phi_i = 40^\circ$, $\epsilon_f = 12.8$, and $d = 3$ cm.

In (4), the transition matrix $[T(d)]$ can be written as

$$[T(d)] = e^{\lambda_1 d}[A_1] + e^{\lambda_2 d}[A_2] \tag{7}$$

where the singular matrices $[A_1]$ and $[A_2]$ do not contain any term that grows exponentially

$$[A_1] = [\tilde{\phi}]\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}[\tilde{\phi}]^{-1} \tag{8}$$

and

$$[A_2] = [\tilde{\phi}]\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{(\lambda_3 - \lambda_2)d} & 0 \\ 0 & 0 & 0 & e^{(\lambda_4 - \lambda_2)d} \end{bmatrix}[\tilde{\phi}]^{-1}. \tag{9}$$

Note that $[A_1]$ is obtained from (4) by replacing the terms of $e^{\lambda_2 d}$, $e^{\lambda_3 d}$, and $e^{\lambda_4 d}$ with 0 and replacing $e^{\lambda_1 d}$ with 1. Since $[A_1]$ is a singular matrix with rank one, it can be shown that

$$[A_1]\tilde{\psi}(0^-) = (\alpha\tilde{c} + \beta\tilde{d})\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \tag{10}$$

where α, β, ($\alpha \neq \beta$) and a_i with $i = 1, 2, 3,$ or 4 are associated with the eigenvectors and are found numerically. In order to overcome the overflow problem, the following variable transformations are defined

$$(\alpha\tilde{c} + \beta\tilde{d})e^{\lambda_1 d} = u \tag{11}$$

and

$$(\tilde{c} + \tilde{d})e^{\lambda_2 d} = v \tag{12}$$

where u and v are the new variables replacing $\tilde{c}$ and $\tilde{d}$. With the variable transformations, we have

$$[T(d)][\tilde{\psi}(0^-)] = u\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} + \frac{u}{\alpha-\beta}[A_2]e^{(\lambda_2-\lambda_1)d}\begin{bmatrix} -j\sqrt{k^2-k_0^2} \\ -\omega\epsilon_0 \\ j\sqrt{k^2-k_0^2} \\ -\omega\mu_0 \end{bmatrix} + \frac{v}{\alpha-\beta}[A_2]\begin{bmatrix} j\beta\sqrt{k^2-k_0^2} \\ \alpha\omega\epsilon_0 \\ -\alpha j\sqrt{k^2-k_0^2} \\ \beta\omega\mu_0 \end{bmatrix}. \tag{13}$$

The new formulation in (13) assures that the matrix equations in (6) is well-behaved and always invertable numerically. As a result, the overflow and singularity problem in finding the spectral electromagnetic fields is overcome.

III. An Example· Scattering from a Biased Ferrite Layer

A practical example of the case of scattering from a biased ferrite layer is shown in Fig. 1. It is known that the biased ferrites may support extraordinary waves [12]. The extraordinary wave is an envanescent wave. When the decay factor of this evanescent wave is large, the transition matrix cascade method is no longer adequate. The results of the reflection from a biased ferrite layer is shown in Fig. 1 for both methods. It is seen that there exists a frequency band where ordinary transition matrix method provides ridiculous results. Outside this frequency band, two methods produce identical results. The asymptotic form of the eigenvalue corresponding to an extraordinary wave, when $\mu \approx 0$, is found as

$$\lambda = \pm\frac{1}{\sqrt{\mu}}\sqrt{k_x^2 + \epsilon_f \kappa^2} \tag{14}$$

where μ and κ are the common notations for the elements of the ferrite permeability tensor. It is seen from (14) that when μ is near zero, $(0 < \mu)$ corresponds to a large positive eigenvalue and $(\mu < 0)$ corresponds to a large purely imaginary eigenvalue. If $\mu = 0$ occurs at f_0, for frequency a little less than f_0, the waves in ferrite are highly oscillatory. Also, when the frequency is a little larger than f_0, large real eigenvalue results in the failure of transition matrix cascade method due to the round-off errors and the singularity of the transition matrix. The computations are carried out in a PC-486 model with Lahey Fortran 77 compiler.

All the numbers are double precision. The run time for each data point is less than a tenth second.

IV. Conclusion

A numerical algorithm was developed for the computation of electromagnetic fields in a generalized anisotropic structure. The proposed method using variable transformation overcomes the difficulty frequently encountered in the transition matrix cascade method, without increasing computational time or memory. The method discussed in this paper may be extended to deal with multi-layered structures.

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